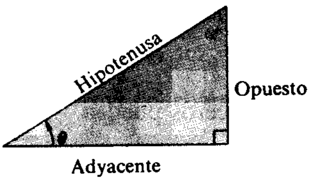
TRIGONOMETRÍA

Definición de las seis funciones trigonométricas

Definición por triángulos rectángulos, con $0 < \theta < \pi/2$.

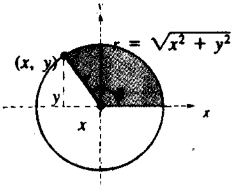


$$\text{sen}\ \theta = \frac{\text{op.}}{\text{hip.}} \qquad \text{cosec}\ \theta = \frac{\text{hip.}}{\text{op.}}$$

$$\cos\ \theta = \frac{\text{ady.}}{\text{hip.}} \qquad \sec\ \theta = \frac{\text{hip.}}{\text{ady.}}$$

$$\text{tg}\ \theta = \frac{\text{op.}}{\text{ady.}} \qquad \text{ctg}\ \theta = \frac{\text{ady.}}{\text{op.}}$$

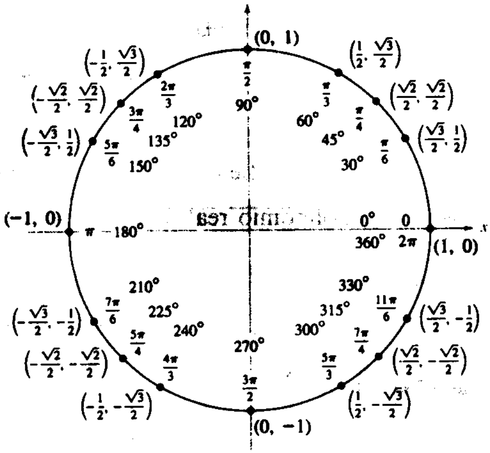
Definición como funciones circulares, para ángulos θ arbitrarios.



$$\text{sen}\ \theta = \frac{y}{r} \qquad \text{cosec}\ \theta = \frac{r}{y}$$

$$\cos\ \theta = \frac{x}{r} \qquad \sec\ \theta = \frac{r}{x}$$

$$\text{tg}\ \theta = \frac{y}{x} \qquad \text{ctg}\ \theta = \frac{x}{y}$$



Identidades recíprocas

$$\text{sen}\ x = \frac{1}{\text{cosec}\ x} \qquad \sec\ x = \frac{1}{\cos\ x} \qquad \text{tg}\ x = \frac{1}{\text{ctg}\ x}$$

$$\text{cosec}\ x = \frac{1}{\text{sen}\ x} \qquad \cos\ x = \frac{1}{\sec\ x} \qquad \text{ctg}\ x = \frac{1}{\text{tg}\ x}$$

Identidades de tangente y cotangente

$$\text{tg}\ x = \frac{\text{sen}\ x}{\cos\ x} \qquad \text{ctg}\ x = \frac{\cos\ x}{\text{sen}\ x}$$

Identidades de Pitágoras

$$\text{sen}^2\ x + \cos^2\ x = 1$$

$$1 + \text{tg}^2\ x = \sec^2\ x \qquad 1 + \text{ctg}^2\ x = \text{cosec}^2\ x$$

Identidades de cofunciones

$$\text{sen}\left(\frac{\pi}{2} - x\right) = \cos\ x \qquad \cos\left(\frac{\pi}{2} - x\right) = \text{sen}\ x$$

$$\text{cosec}\left(\frac{\pi}{2} - x\right) = \sec\ x \qquad \text{tg}\left(\frac{\pi}{2} - x\right) = \text{ctg}\ x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \text{cosec}\ x \qquad \text{ctg}\left(\frac{\pi}{2} - x\right) = \text{tg}\ x$$

Fórmulas de reducción

$$\text{sen}\ (-x) = -\text{sen}\ x \qquad \cos\ (-x) = \cos\ x$$

$$\text{cosec}\ (-x) = -\text{cosec}\ x \qquad \text{tg}\ (-x) = -\ \text{tg}\ x$$

$$\sec\ (-x) = -\sec\ x \qquad \text{ctg}\ (-x) = -\text{ctg}\ x$$

Fórmulas de suma y diferencia

$$\text{sen}\ (u \pm v) = \text{sen}\ u \cos\ v \pm \cos\ u\ \text{sen}\ v$$

$$\cos\ (u \pm v) = \cos\ u \cos\ v \mp \text{sen}\ u\ \text{sen}\ v$$

$$\text{tg}\ (u \pm v) = \frac{\text{tg}\ u \pm \text{tg}\ v}{1 \mp \text{tg}\ u\ \text{tg}\ v}$$

Fórmulas del ángulo doble

$$\text{sen}\ 2u = 2\ \text{sen}\ u \cos\ u$$

$$\cos\ 2u = \cos^2 u - \text{sen}^2 u = 2\cos^2 u - 1 = 1 - 2\,\text{sen}^2 u$$

$$\text{tg}\ 2u = \frac{2\ \text{tg}\ u}{1 - \text{tg}^2\ u}$$

Fórmulas de reducción de potencias

$$\text{sen}^2\ u = \frac{1 - \cos\ 2u}{2}$$

$$\cos^2\ u = \frac{1 + \cos\ 2u}{2}$$

$$\text{tg}^2\ u = \frac{1 - \cos\ 2u}{1 + \cos\ 2u}$$

Fórmulas suma-producto

$$\text{sen}\ u + \text{sen}\ v = 2\ \text{sen}\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\text{sen}\ u - \text{sen}\ v = 2\cos\left(\frac{u + v}{2}\right)\text{sen}\left(\frac{u - v}{2}\right)$$

$$\cos\ u + \cos\ v = 2\cos\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\cos\ u - \cos\ v = -2\ \text{sen}\left(\frac{u + v}{2}\right)\text{sen}\left(\frac{u - v}{2}\right)$$

Fórmulas producto-suma

$$\text{sen}\ u\ \text{sen}\ v = \frac{1}{2}\left[\cos\ (u - v) - \cos\ (u + v)\right]$$

$$\cos\ u \cos\ v = \frac{1}{2}\left[\cos\ (u - v) + \cos\ (u + v)\right]$$

$$\text{sen}\ u \cos\ v = \frac{1}{2}\left[\text{sen}\ (u + v) + \text{sen}\ (u - v)\right]$$

$$\cos\ u\ \text{sen}\ v = \frac{1}{2}\left[\text{sen}\ (u + v) - \text{sen}\ (u - v)\right]$$